



# Colloquium del Departamento de Análisis Matemático

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**“A Perron-Frobenius Operator that Intertwines  
the Hilbert transform: An application to the  
Klein-Gordon Equation”**

**3 de Abril de 2014**

a las 13:00 horas en el seminario 222

**Abstract:**

Hedenmalm and Montes-Rodríguez have shown that if the Fourier transform of an absolutely continuous measure vanishes on certain lattice-cross then the Fourier transform is the zero function if and only if the space between the points in the lattice is less than or equal to 1. The latter problem is equivalent to show that the exponentials  $\exp(inx)$  and  $\exp(in/x)$ , with  $n$  and  $m$  integers, span a weakly dense subspace in the space of essential bounded functions on the real line.

However, it remained open the problem if the same exponentials, now with  $n$  and  $m$  running only on the positive integers, span a weakly dense subspace of space of bounded functions that extend via the Poisson Kernel to a holomorphic function on the upper half-plane. The latter problem is equivalent to the fact that if a solution, which is the Fourier transform of an absolutely continuous measure, of the Klein-Gordon equation vanishes on the positive integers, then it also vanishes on the first quadrant. Recently, we have been able to solve this problem in the affirmative.

The proof is based on an analysis of how the Hilbert transform intertwines with certain Perron-Frobenius operator. There are several consequences. For instance the algebra  $A$  spanned by certain classical inner functions is weakly dense in the space of bounded holomorphic functions on the unit disk.

Joint work with H. Hedenmalm from KTH (Stokholm)