## Conferencia Santaló 2023

La Facultad de Ciencias Matemáticas y la Revista Matemática Complutense organizan anualmente la Conferencia Santaló, en memoria de Luis Santaló (1911-2001), doctor por esta universidad. La conferencia es impartida cada año por un/a profesor/a de reconocido prestigio. La conferenciante de este curso es:

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## The role of Liouville-type theorems in partial differential equations

The Cauchy-Liouville theorem (1844) states that any bounded entire function of a complex variable is necessarily constant. Another related and very classical fact is that this remains true for any harmonic function over the whole Euclidean space (in arbitrary dimensions). In the realm of PDE's, by a Liouville-type theorem, one usually means a statement asserting the nonexistence of solutions in the whole space (or a suitable unbounded domain), the solutions being sometimes subjected to certain restrictions (e.g., nonconstant, or with some sign or growth conditions). Numerous results of this kind have appeared over the years and many far-reaching applications have arisen, conferring Liouville-type theorems an important role in the theory of PDE's. In this lecture, we will survey some aspects of historical and current developments on the topic, and underline some connections with other mathematical areas, such as the calculus of variations, geometry, fluid dynamics or optimal stochastic control. Starting with the problem of minimal surfaces (Lagrange, Bernstein, de Giorgi, Bombieri,...) and with the connections of Liouville-type theorems with regularity theory for linear elliptic systems (Giaquinta, Nečas,...), we will then turn to semilinear elliptic equations (where strong initial motivation came from the Yamabe problem), eventually leading to the development (1980-2000's) of powerful tools to show existence and a priori estimates for nonlinear Dirichlet problems. In connection with Liouville-type theorems, we will encounter such key concepts as scaling, zooming and doubling techniques, Alexandrov-Serrin moving planes methods and topological degree.

In the more recent period, this line of research has also led to much progress in the study of singularities of solutions, both for stationary (elliptic) and evolution PDEs. In the latter category, especially for the description of finite time blow-up singularities, Liouville-type theorems turn out to play an important role in numerous problems (nonlinear reaction-diffusion, Hamilton-Jacobi and Navier-Stokes equations, harmonic maps and Ricci flows, KdV, nonlinear wave and Schrödinger equations,...). In view of such diversity, we will thus of course give only a partial overview.



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